Name, ID

$t\langle 1,-3,2\rangle+(2,0,1)$

Exam I: MTH 111, Spring 2017

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Points = $\frac{\cancel{53}\ 58}{58}$

QUESTION 1. (4 points) Given that the line $L = 2 + t, y = -3t, z = 1 + 2t$ is perpendicular to a plane, say P. If the point $(1, 2, -5)$ lies in the plane P, find the equation of the plane P.

The parametric eqn can be written as $L: t\langle 1,-3,2\rangle + (2,0,1)$

since $L \perp$ to plane & pt $(1,2,-5)$ lies on the plane

$1(x-1) + -3(y-2) + 2(z+5) = 0$

$x - 1 - 3y + 6 + 2z + 10 = 0$

$x - 3y + 2z + 15 = 0$

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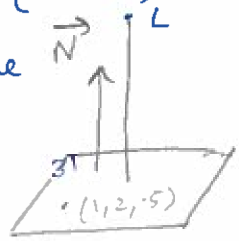

QUESTION 2. (5 points) The two planes $P_1 : 2x - y + z = 6$ and $P_2 : -x + y + 4z = 4$ intersect in a line L. Find a parametric equations of L.

$P_1 : 2x - y + z = 6 \quad \langle 2,-1,1\rangle \rightarrow \vec{N_1}$

$P_2 : -x + y + 4z = 4 \quad \langle -1,1,4\rangle \rightarrow \vec{N_2}$

$$\vec{N_1} \times \vec{N_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ -1 & 1 & 4 \end{vmatrix} = \hat{i}(-4-1) - \hat{j}(8+1) + \hat{k}(2-1)$$

$= -5\hat{i} - 9\hat{j} + \hat{k} \rightarrow \langle -5,-9,1\rangle$

Assume $z = 0$

$2x - y = 6$

$+ \quad -x + y = 4$

$x = 10$

$-10 + y = 4$

$y = 14$

pt $(10,14,0)$

$L : t\langle -5,-9,1\rangle + (10,14,0)$

$: \langle -5t, -9t, t\rangle + (10,14,0)$

$x = -5t + 10 \; ; \; y = -9t + 14 \; ; \; z = t$

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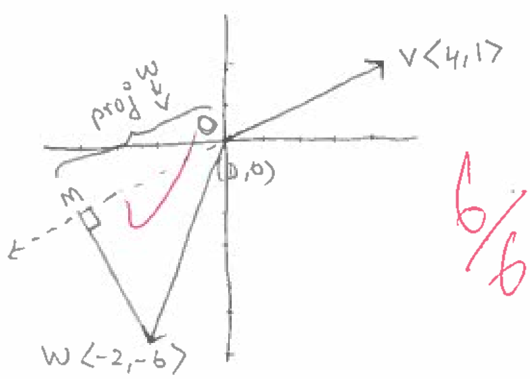
QUESTION 3. (6 points) From the origin (i.e, (0, 0)) draw the two vectors $V = \langle 4, 1 \rangle$, $W = \langle -2, -6 \rangle$. First draw $Proj_V^W$. Then find $Proj_V^W$ and its length.

$$\text{proj}_V^W = \frac{V \cdot W}{|V|^2} \cdot V = \frac{-8-6}{17}\langle 4,1\rangle$$

$$= \frac{-14}{17}\langle 4,1\rangle$$

$$= \left\langle \frac{-56}{17}, \frac{-14}{17} \right\rangle$$



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$$|\text{proj}_V^W| = \sqrt{\left(\frac{-56}{17}\right)^2 + \left(\frac{-14}{17}\right)^2}$$

$$= \sqrt{11.52} = 3.39$$

3136
+ 196
3332
289
11.52

QUESTION 4. (3 points) Given that $y = -2$ is the directrix of a parabola that has focus F. If the point $Q = (4, 7)$ lies on the curve of the parabola, find $|QF|$ (i.e., find the distance between F and Q).

$|QL| = 9$

since $|QF| = |QL|$

$\therefore |QF| = 9$ units

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(diagram on next page)

QUESTION 5. (8 points) Given $(-4, 2), (6, 2), (1, 5)$ are three vertices of an ellipse.

(i) Find the fourth vertex of the ellipse (you may want to draw such ellipse).

since centre = $(1, 2)$ $|CV_3| = |CV_4| = 3$

$V_4 (1, -1)$ //

(ii) Find the ellipse-constant K.

~~$F_1 F_2$~~ $= |V_1 V_2| = 10 = K$

(iii) Find the Foci of the ellipse.

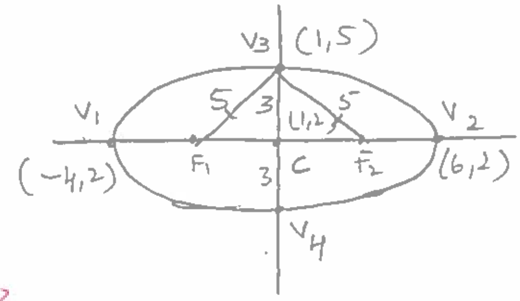
$|V_3 F_1| = |V_3 F_2| = K/2 = 5$

$|F_1 C| = 4$

$F_1(-3, 2)$ $F_2(5, 2)$

(iv) Find the equation of the ellipse.

$$\frac{(x-1)^2}{25} + \frac{(y-2)^2}{9} = 1$$



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$\frac{-4+6}{2}$

centre $(1, 2)$

$\frac{x+1}{2} = 1$ $\frac{5+y}{2} = 2$

$x + 1 = 2$ $5 + y = 4$

$x = 1$ $y = -$

QUESTION 6. (6 points) Given $F_1 = (0, -3)$, $F_2 = (0, 1)$ are the foci of an ellipse and $v = (0, 3)$ is one of the vertices. Find the ellipse-constant K and the equation of the ellipse.

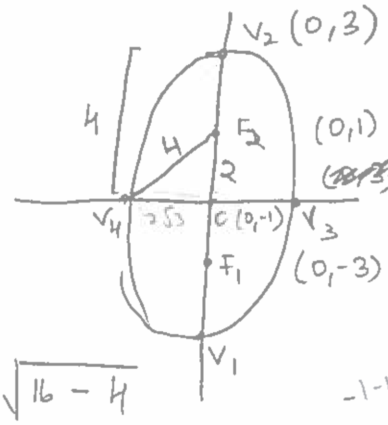
$\therefore$ centre $(0, -1)$

$|CV_2| = 4$ units

$V_1(0, -5)$

$\therefore$ ellipse constant $= |V_1 V_2| = 8 = K$

$$\frac{x^2}{12} + \frac{(y+1)^2}{16} = 1$$



$\frac{1-3}{2}$ $\frac{-2}{2}$

$CV_4 = \sqrt{16 - 4}$

$= \sqrt{12} = 2\sqrt{3}$

$-1-1$

2×6

$2 \times 2 \times 3$

$2\sqrt{3}$

$4 \times 3 = 12$

QUESTION 7. (8 points) First draw the hyperbola $\frac{y^2}{4} - \frac{(x-1)^2}{12} = 1$. Then find

a) The hyperbola-constant K.

$\left(\frac{K}{2}\right)^2 = 4$ $\frac{K}{2} = 2$ $K = 4$

b) The two vertices of the hyperbola.

$V_1(1, 2)$ //

$V_2(1, -2)$ //

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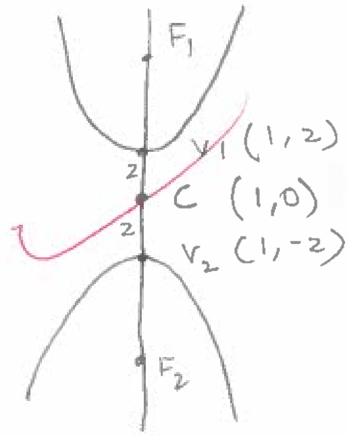
$b^2 = 12$

$b = \sqrt{12}$

$= 2\sqrt{3}$



c) The foci of the hyperbola.

$F_1 C = \sqrt{b^2 + (4/2)^2} = $ ~~$\sqrt{12+64}$~~ $=$ ~~$12+64$~~ $\sqrt{12 + 4} = \sqrt{16}$

$= 4$

$F_1(1, 4)$ //

$F_2(1, -4)$ //

QUESTION 8. (6 points) Given $x = -4$ is the directrix of of a parabola that has the point $(6,5)$ as its vertex point.

a) Find the equation of the parabola

$4(10)(x-6) = (y-5)^2$

$= 40(x-6) = (y-5)^2$

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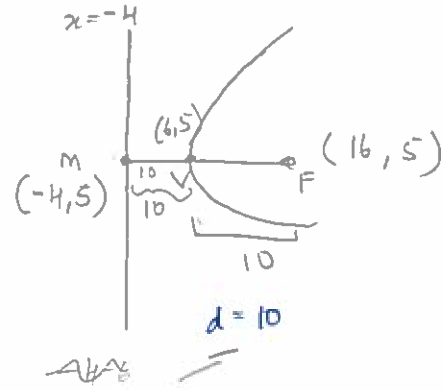

b) Find the focus of the parabola.

$F(16, 5)$

QUESTION 9. (6 points) Consider the parabola $x = -0.25(y+3)^2 + 4$ [hint: first write it in the standard form].

$x = -0.25(y+3)^2 + 4$

$(x-4) = -0.25(y+3)^2$

$-4(x-4) = (y+3)^2$

$\cancel{4}d = -\cancel{4}$

$d = -1$

x=5

$x = 5$

a) Find the focus.

~~F(5,-3)~~ $F = (3,-3)$

b) Find the equation of the directrix

$x = 5$

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c) Draw the parabola

QUESTION 10. (6 points) Given two lines $L_1: x = t, y = 1+t, z = 3-2t$, $L_2: x = 2+w, y = 3-w, z = -1+2w$. If L_1 intersects L_2, find the intersection point.

$L_1:\ x = t$
$y = 1+t$
$z = 3-2t$

$L_2:\ x = 2+w$
$y = 3-w$
$z = -1+2w$

$L_1:\ x = 2$
$y = 3$
$z = -1$

$L_2:\ x = 2$
$y = 3$
$z = -1$

$t = 2 + w$

$t - w = 2$ — (1) ×2

$1 + t = 3 - w$

$t + w = 3 - 1$

$t + w = 2$ — (2) ×2

$3 - 2t = 2w - 1$

$2w + 2t = 4$ — (3)

~~2w+2t=4~~
~~2w~~

$2w + 2t = 4$
~~$-2w+2t =$~~

$t - w = 2$
$+\ t + w = 2$

$2t = 4$

$t = 2$ $w = 0$

∴ The point of intersection is $(2, 3, -1)$

QUESTION 11. Bonus: (4 points) Imagine this: You are staring at 4 tables; table one has 3 legs; table 2 has 4 legs; table 3 has 6 legs; table 4 has 8 legs. Which one of the tables is more stable? explain CLEARLY and briefly in order to get the full mark (NO PARTIAL CREDIT, i.e., 0 or 4)

The table 4 with 8 legs is more stable. since there are 8 legs ~~each~~ The weight on the table will be equally distributed among more number of legs. ∴ ~~Each leg~~ will have to support less weight as compared to table 1 when each leg will have to support more weight.

Faculty information

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